Hiccups for HIPCs?*

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Abstract

In this paper we discuss monetary and fiscal policy issues facing heavily-indebted poor countries (HIPCs) who receive debt reduction via the enhanced HIPC initiative. This debt relief program is distinguished from previous ones by its conditionality: freed resources must be used for poverty reduction. We argue that (i) this conditionality limits the extent to which the initiative relaxes the government's lifetime budget constraint; (ii) depending on the response of monetary policy to an increase in social spending there could be a short-run increase in inflation in HIPC countries and (iii) the keys to long-run fiscal sustainability in the HIPCs are significant fiscal reforms by their governments, and the effectiveness of their poverty reduction programs in raising growth.

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The enhanced HIPC initiative has acquired increasing prominence in the policy debate in developing countries.[1] A significant number of countries are already enjoying debt relief under the HIPC initiative. While an abundant literature developed during the 1980s on the macroeconomic impact of debt overhang and debt write-offs, little attention has been devoted to the macroeconomic impact of debt relief proposed under the enhanced HIPC initiative.[2] Debt relief under the HIPC initiative differs from previous major debt relief initiatives, such as the Baker and Brady plans, in that it concerns official rather than commercial debt and in that it imposes well-defined conditionality. In particular, it requires that budgetary resources no longer needed for debt service be used for poverty reduction purposes. This also distinguishes the enhanced HIPC initiative from previous official debt relief programs.

We focus on monetary and fiscal policy issues connected with debt reduction under the enhanced HIPC initiative. The issues we highlight stem from two distinguishing aspects of the initiative's design. First, the HIPC initiative relieves debt through forgiveness of a substantial fraction of a country's debt service payments. The initiative requires that the resources freed from debt service be used to increase government spending on poverty reduction programs.[3] Second, the initiative has a finite life—the increase in government spending and the forgiven debt service take place over a floating period with length depending on the country's success in implementing a comprehensive anti-poverty strategy.[4] Given these features of the HIPC initiative we make three main points.[5]

First, we argue that the conditionality of HIPC debt relief limits the extent to which the initiative relaxes the government's lifetime budget constraint.[6] The reason for this is fairly straightforward: the government is swapping one type of spending commitment for another of equal value. Second, we argue that monetary policy makers face a trade-off between stabilizing inflation and achieving permanent debt reduction. This trade-off is related to the

[1] HIPC is an acronym for heavily-indebted poor country.

[2] See, among many others, Sachs (1984, 1989), Frenkel, Dooley and Wickam (1989), Claessens, Diwan and Fernandez-Arias (1992) and Fernández-Arias (1992).

[3] For a concise description of the HIPC initiative see Van Trotsenburg and MacArthur (1999) and the World Bank's HIPC website: www.worldbank.org/hipc.

[4] After a country qualifies for assistance, the international community commits debt relief to reach a target for the ratio of the net-present value of debt to exports (150 percent) or to government revenue (250 percent). This assistance is delivered over a variable period during which there is conditionality requiring increased poverty reduction spending. After a country has implemented a comprehensive poverty reduction strategy, creditors supply any remaining debt relief needed to reach sustainability without further conditionality.

[5] In what follows we will frequently refer to the enhanced HIPC initiative as "the initiative," its characteristic conditionality as "HIPC conditionality," governments and organizations providing debt relief as "donors" and countries receiving debt relief as "recipient countries," or simply "HIPCs."

[6] For an early analysis of the financial impact of the HIPC initiative, see World Bank (2001).

standard one faced by any central bank when confronted by a temporary increase in domestic demand. Third, we discuss two factors that mitigate our first two points. If the poverty reduction programs, initiated under debt relief, substantially increase economic activity, governments will reap additional tax revenue that will help finance them. Furthermore, if recipient governments act on their own to implement significant additional fiscal reforms, they are more likely to attain long-run fiscal sustainability. Hence, we argue that poverty reduction programs with a significant growth enhancing component, and significant fiscal reforms by the recipient governments will be the true keystones of success.

Our analysis does not focus on assessing the effectiveness of poverty reduction spending in reducing poverty, rather we focus our analysis on the budgetary impact of the HIPC initiative. However, the impact of this spending on output is important in our analysis because of its secondary impact on tax revenue. To highlight the role of different aspects of the initiative, we decompose its effect on the government budget into two components: the direct effect, which ignores effects on tax revenue, and the indirect effect, which takes them into account. In measuring the indirect effect, we take as given Burnside and Dollar's (2000) results suggesting that aid is effective in raising growth in recipient countries with good macroeconomic policies. Thus our results give a plausible upper bound on the positive *budgetary* impact of the HIPC initiative.

In Section 1 we focus on our first point related to the relaxation of the government budget constraint. We illustrate the impact of debt relief with HIPC-conditionality on a government's finances using a standard model of the government budget constraint. This model only allows us to discuss the direct effects of debt relief with HIPC-conditionality on the budget, which we argue are limited. We argue that other things equal, after debt relief, the government must still raise the same amount of revenue, from all sources, as it did prior to receiving debt relief.

In Section 2 we develop a simple monetary model based on the standard Cagan money demand function, in order to fully characterize the equilibrium dynamics of prices, inflation, debt and seigniorage during and after the implementation of a debt relief initiative. In Section 3 we simulate our model under different monetary policies. By doing this we are able to highlight our second point. We find that, with what we describe as passive monetary policy, debt-relief with HIPC conditionality could have a short-term impact on money creation and inflation, which would eventually be reversed. Under what we describe as a more active monetary policy, the government can act to stabilize inflation, but it can only do so by raising its long-run indebtedness relative to what it would be under passive policy. This is because keeping inflation stable requires the government to issue more new debt to sterilize the monetary impact of the short-run increase in government spending under HIPC-conditionality.

Section 3 also highlights the role of the mitigating factors mentioned above: we show that any growth resulting from debt relief and any additional fiscal reforms implemented by a HIPC's government, would both (i) tend to improve its lifetime budget constraint, and (ii) lessen the short-run monetary impact of the increase in its spending. For these reasons, we suggest that the two essential ingredients for lasting *fiscal sustainability* in HIPC countries are (i) well-designed and effective growth-enhancing poverty reduction initiatives, and (ii) significant fiscal reforms that are not met by reduced aid commitments from donors.

In Section 4 we discuss some shortcomings of and caveats to our analysis and answer some frequently asked questions. In Section 5 we discuss possible extensions to our model, and provide some concluding remarks.

1. The Government Budget Constraint and HIPC Conditionality

We begin by discussing the fiscal implications of debt relief with "HIPC-conditionality" within a standard model of the government's intertemporal budget constraint. In Appendix A we show that this approach is equivalent to working with the simple static accounting framework familiar to students of monetary theory and policy.

1.1. The Government Budget Constraint: *A Baseline Interpretation*

We now present a standard model of the government's intertemporal budget constraint in continuous time. In our simple model, there is only one good, whose price is P_t. The government issues only one type of debt, D_t, whose value is indexed in terms of that good. Thus we eliminate implicit default, through unanticipated inflation, by assumption. An important further aspect of our baseline example is that we also rule out explicit default, i.e. the government always meets its debt service obligations. In a later subsection, we describe an alternative example which allows for explicit default. In a third subsection we provide a further interpretation that distinguishes between pre-existing debt and new debt.

We assume, for simplicity, that the net real interest rate on government debt is some constant r. The government finances its interest payments, rD_t, and its purchases of goods and services, G_t, in four ways: by raising tax revenue (net of transfers), Ω_t, through the issuance of base money, M_t, by receiving aid, A_t, or through the issuance of new debt. The government raises funds by issuing base money via seigniorage revenue, $\dot{M}_t/P_t$, where P_t is the price level and $\dot{M}_t$ is the time derivative of the money stock.[7] Hence, the government's flow budget constraint is given by

$$\dot{D}_t = rD_t + G_t - \Omega_t - A_t - \dot{M}_t/P_t, \tag{1.1}$$

where all variables are measured in units of local currency.

Writing the budget constraint in terms of D_0 and iterating, we obtain

$$D_0 = \int_0^\infty (\Omega_t - G_t + A_t + \dot{M}_t/P_t)e^{-rt}dt \tag{1.2}$$

where we have imposed the no-Ponzi scheme condition that

$$\lim_{t\to\infty} e^{-rt}D_t = 0.$$

We want to interpret the HIPC initiative using equations (1.1) and (1.2). Our starting point is to think of a working definition of a heavily indebted government. One interpretation of a heavily indebted government at time 0 is as follows: given the initial stock of debt, D_0, and the likely paths of government purchases and foreign aid receipts, $\{A_t, G_t\}_{t\in[0,\infty)}$, the combined present values of taxes and seigniorage revenue required to close the government budget constraint is very large, i.e.:

$$D_0 + \int_0^\infty (G_t - A_t)e^{-rt}dt = \int_0^\infty [\Omega_t + (\dot{M}_t/P_t)]e^{-rt}dt \gg 0.$$

In other words, we could think of HIPC governments as ones which, in order to be solvent, would need to rely on either (i) significant future seigniorage revenue, obtained at the cost of high inflation, or (ii) punitively high future tax revenues.

Our next step is to see what the impact of the HIPC initiative would be on the government's lifetime budget constraint. To do this, we must characterize the initiative in terms of its effect on the various items in the government's lifetime budget constraint (1.2).

[7] We generically indicate time derivatives, $\partial Z_t/\partial t$, as $\dot{Z}_t$.

Direct Effects

As we described in the introduction, one feature of the HIPC initiative is that donors forgive some fraction of the scheduled debt service payments pertaining to a government's existing stock of external debt. At the decision point, which we will refer to as date 0, the present value of the existing scheduled debt service is calculated, and is expressed as a percentage of the country's exports. The debt relief given under the HIPC initiative would forgive enough of this debt service to reduce this *NPV of debt-to-exports ratio* to no more than 150 percent.[8] To take an example, suppose the NPV of debt-to-exports ratio at date 0 was 260 percent.[9]. Then, over the life of the initiative, the country would reduce its service payments, *on debt existing at time 0*, by about 38 percent in present value terms.

There are several ways we could build this kind of debt relief into equations (1.2) and (1.1). One way would be to assume that before debt relief, D_0, on the left-hand side of (1.2), takes on some value. We could then assume that after debt relief, there is simply a change in the left-hand side of (1.2), to a new value $D_0' = (1-\theta)D_0$, where θ represents the fraction of the country's debt that is effectively cancelled by the forgiven debt service.

An alternative interpretation is that at date 0 the country receives an announcement from donors stating that the present value of the future path of $\{A_t\}_{t\in[0,\infty)}$ will be higher by the amount θD_0 $(0 < \theta < 1)$, than it would have been in the absence of the initiative.

Notice that these two interpretations are isomorphic to one another in terms of their implications for the remaining items in the government's lifetime budget constraint. In particular, under the first interpretation the initial stock of debt is $(1-\theta)D_0$. Holding the present value of aid flows constant, this implies that $\int_0^\infty (\Omega_t - G_t + \dot{M}_t/P_t)e^{-rt}dt$ will fall by θD_0 under the debt-relief initiative. On the other hand, under the second interpretation the initial stock of debt is D_0, and the present value of future aid flows rises by θD_0. Again, this implies that $\int_0^\infty (\Omega_t - G_t + \dot{M}_t/P_t)e^{-rt}dt$ will fall by θD_0 under the initiative. We find the latter interpretation to be more convenient notationally, and use it throughout the rest of this paper.

The next important feature of the HIPC initiative is its conditionality, requiring that the savings from reduced debt service be used to increase social spending. We model this

[8] In some cases debt relief is calculated with reference to the present value of debt service relative to government revenue.

[9] This was the average figure across the 22 HIPCs that had reached the decision date by 1999. See Development Committee (2001).

conditionality as being equivalent to the government making an announcement at date 0 that the present value of the future path of $\{G_t\}_{t\in[0,\infty)}$ will be higher by the amount θD_0.

We refer to the effects of the HIPC initiative on the paths of A_t and G_t as its *direct* effects. We have characterized these effects as a change in the anticipated paths of aid and government spending to $\{A_t', G_t'\}_{t\in[0,\infty)}$ from $\{A_t, G_t\}_{t\in[0,\infty)}$, where

$$\int_0^\infty A_t' e^{-rt} = \int_0^\infty A_t e^{-rt} + \theta D_0 \tag{1.3}$$

$$\int_0^\infty G_t' e^{-rt} = \int_0^\infty G_t e^{-rt} + \theta D_0. \tag{1.4}$$

Since (1.2) must be satisfied for all possible future paths, we have

$$D_0 = \int_0^\infty (\Omega_t' - G_t' + A_t' + \dot{M}_t'/P_t')e^{-rt}dt, \tag{1.5}$$

where $\{\Omega_t', M_t', P_t'\}_{t\in[0,\infty)}$ are the post-debt relief paths of taxes, the money supply and the price level. Given (1.3) and (1.4) we can rewrite (1.5) as

$$D_0 = \int_0^\infty (\Omega_t' + \dot{M}_t'/P_t')e^{-rt}dt + \int_0^\infty (A_t - G_t)\, e^{-rt}dt.$$

It is clear from (1.2), however, that this implies

$$\int_0^\infty (\Omega_t' + \dot{M}_t'/P_t')e^{-rt}dt = \int_0^\infty (\Omega_t + \dot{M}_t/P_t)e^{-rt}dt, \tag{1.6}$$

where $\{\Omega_t, M_t, P_t\}_{t\in[0,\infty)}$ are the paths of taxes, the money supply and the price level that would have prevailed in the absence of debt relief.

It is clear, from (1.6), that debt relief with HIPC conditionality does not relax the government budget constraint in the following sense. To satisfy its budget constraint, the government must raise just as much seigniorage and tax revenue after receiving debt relief as it needed to in the absence of debt relief.

In section 3 we will consider a case where $\int_0^\infty \Omega_t' e^{-rt}dt = \int_0^\infty \Omega_t e^{-rt}dt$, i.e. there is no change in tax revenues induced by the initiative. In this setting, (1.6) implies that the initiative can have no impact on the present value of seigniorage revenue. On the other hand, depending on how monetary policy responds to increased government spending, and depending on the timing of that increased spending, the inflation rate can rise, fall, or remain unchanged in the short-run. We will see, later, in some model-based experiments, that if there is a temporary rise in inflation, it will later fall below its initial value. If, for some reason, inflation were to decline in the short run, it would rise above its initial value in the long-run.[10] On the other hand, it is possible that inflation could remain unchanged.[11] To achieve a desired path for inflation, the government must choose the appropriate monetary policy. In section 4 we will see that the monetary policy consistent with a stable inflation path is an active one–in the sense that the central bank must neutralize the monetary injection resulting from increased government spending.

Indirect Effects

Of course, eventually we must modify our analysis to take account of the indirect effects of debt relief on the budget. We will do this in detail in sections 3 and 4, but here we simply note where that analysis will lead us. Suppose that the increased social spending required under the HIPC initiative has a positive impact on growth in recipient countries. In this case, there will be some relaxation of the government's budget constraint. It will still be true that the combined present values of taxes and seigniorage revenue will be unchanged. I.e., (1.6) will still hold. However, now suppose the government leaves tax *rates* unchanged when it receives debt relief. In this case, the present value of tax revenue will rise, allowing the present value of seigniorage revenue to be lower. Furthermore, if there is an increase in money demand resulting from an economic expansion, this seigniorage revenue could be raised at a lower steady state rate of inflation. Thus, if we measured the government's fiscal health by how much inflation it would need to generate to close its budget, both of these effects would be beneficial. In order to quantify these indirect effects we need a fully specified model, such as the one we present in section 3.

1.2. The Government Budget Constraint: *An Alternative Interpretation*

In this section we modify our analysis of the government budget constraint to explicitly allow for default, but we will maintain our assumption that debt is denominated in real terms. What we mean by explicit default is that the government can announce paths, $\{\Omega_t, G_t, M_t\}_{t\in[0,\infty)}$, that along with the path, $\{A_t\}_{t\in[0,\infty)}$, lead to a violation of (1.2). A version of (1.2) still holds, but in this version the left hand side variable, D_0^M, is a measure of the market value (at the discount rate r) of the government's future primary surpluses

[10] This basic point lies at the core of models of currency crises in which governments that are unable to close their lifetime budget constraints without resort to seignorage revenue are required to eventually abandon any exchange rate arrangement that limits their access to it. See, for example, Krugman (1979), Flood and Garber (1984), Obstfeld (1986), Drazen and Helpman (1987), and Burnside, Eichenbaum and Rebelo (2001).

[11] Anticipating our later results, this would be the case if $M_t' = M_t$ for all t.

inclusive of aid and seigniorage revenue

$$D_0^M = \int_0^\infty (\Omega_t + A_t + \dot{M}_t/P_t - G_t)e^{-rt}dt. \tag{1.7}$$

Essentially, (1.7) recognizes the fact that governments often do not raise sufficient funds to honor (1.2).[12]

This naturally leads to an alternative interpretation of a heavily indebted government: given the government's announced paths for $\{\Omega_t, G_t, M_t\}_{t\in[0,\infty)}$ and the donors' announced path for $\{A_t\}_{t\in[0,\infty)}$, the market value of the government's debt is substantially lower than its face value:

$$D_0^M \ll D_0.$$

Under this interpretation we do not need to think of HIPC governments as ones which would require extraordinarily high future tax or seigniorage revenues to close the budget constraint (1.2). Instead we can think of these governments as ones which are unable (or unwilling) to raise sufficient revenues to do so.

How would we measure the impact of the HIPC initiative in this setting? In the previous subsection, we gave two interpretations of the HIPC initiative. In one interpretation, the initiative directly reduces the initial stock of debt to $D_0' = (1-\theta)D_0$. It also increase the present value of G_t by θD_0. So in this case, the market value of the government future surpluses, D_0^M falls by exactly the decline in initial debt. According to our main interpretation, D_0 doesn't change, but the present values of A_t and G_t both increase by θD_0. Therefore, D_0^M doesn't change either. According to both of these interpretations there is no change in the government's "solvency" as measured by $D_0 - D_0^M$.

1.3. The Government Budget Constraint: *One More Interpretation*

We wish to emphasize that our conclusions regarding the impact of the HIPC initiative on the government's budget are not specific to our modeling the initiative as an increase in aid flows, with those flows being used to make debt service payments. Our results will be unchanged when we introduce separate notation for the debt service payments associated with the initial debt stock, D_0, and we examine a reduction in the magnitude of these

[12] By market value we do not mean to suggest that there is an active market in which the debt of HIPC countries is traded. Rather, by market value, we refer to the value of a claim to the entire future stream of the government's actual primary surpluses inclusive of seigniorage revenue.

payments.[13]

To see this, consider the flow budget constraint, (1.1), which we repeat here:

$$\dot{D}_t = rD_t + G_t - \Omega_t - A_t - \dot{M}_t/P_t,$$

Even when a government doesn't satisfy the lifetime budget constraint, (1.2), this equation describes the evolution of the value of the book value of its debt.[14] We note that for $t > 0$, the term $\dot{D}_t$ can be broken into three components: (i) new debt issued at date t (this can be either new contractual debt or new additions to the stock of debt due to arrears) minus (ii) scheduled amortization of any debt issued *after* date 0, minus (iii) scheduled amortization of debt issued before date 0. The term rD_t can be broken into two pieces: (iv) interest on debt issued *after* time 0, and (v) scheduled interest on debt issued before date 0.

Let the stock of debt issued from date 0 forward be B_t, and notice that $B_0 = 0$. The terms (i) minus (ii) clearly equal $\dot{B}_t$. The term (iv) is rB_t. We let S_t denote the sum of terms (iii) and (v): i.e. scheduled debt service–interest plus amortization–on debt that existed at time 0. So now we can rewrite the government's flow budget constraint as

$$\dot{B}_t = rB_t + S_t + G_t - \Omega_t - A_t - \dot{M}_t/P_t. \tag{1.8}$$

If we roll this budget constraint forward, notice that we obtain

$$0 = B_0 = e^{-r\bar{t}}B_{\bar{t}} + \int_0^{\bar{t}} (\Omega_t + A_t + \dot{M}_t/P_t - G_t - S_t)e^{-rt}dt \tag{1.9}$$

where $\bar{t}$ is the date at which all pre-existing debt at time 0, would have been paid off given the schedule of service payments on that debt. Presumably, $\bar{t} \geq T$, given the initiative's design.

We can model debt relief with HIPC conditionality as a cut in the present value of the stream of debt service payments, $\int_0^{\bar{t}} S_t e^{-rt}dt$, with an increase in the present value of government purchases, $\int_0^{\bar{t}} G_t e^{-rt}dt$, of an equal amount. Clearly, then, this does not allow for a change in the sum of the other items in (1.9), $e^{-r\bar{t}}B_{\bar{t}} + \int_0^{\bar{t}}(\Omega_t + A_t + \dot{M}_t/P_t)e^{-rt}dt$. It is in this sense that the government's lifetime budget constraint is not relaxed. Furthermore, it is clear that we could model the initiative as an increase in the present value of A_t (rather than a decrease in the present value of S_t), and we would reach the same conclusion.

[13] To this point, what is special about our analysis is that we have ignored the indirect effects of the initiative.

[14] Of course, this statement assumes that the interest rate appropriate for the book value is some constant, r.

2. Extending the Model

In this section we extend our model so that we can describe (i) the indirect effects of the HIPC initiative on the government's lifetime budget constraint and (ii) the dynamic inflationary implications of different policy responses to debt relief. Our extensions consist of a money demand function, and a simple model of how debt service savings redirected to poverty reduction spending map into increased economic activity.

2.1. The Money Demand Function

We model the demand for local currency using the familiar Cagan (1956) money demand function

$$\ln(M_t/P_t) = a + \ln Y_t - \eta(r + \pi_t) \tag{2.1}$$

where a is some constant, Y_t represents the level of output, π_t is the inflation rate (i.e. $\pi_t = \dot{P}_t/P_t$) and $\eta > 0$. We do not believe that the implications of the Cagan specification differ substantially, in any qualitative manner, from those derived from alternative monetary models.[15] However, the Cagan specification has computational advantages.

We can use the fact that $\pi_t = \dot{P}_t/P_t$ to derive a generic solution for the price level under the assumption that it is a continuous function of time.[16] We can rewrite (2.1) as

$$p_t = \eta r - a + \ln(M_t/Y_t) + \eta\dot{p}_t, \tag{2.2}$$

where $p_t = \ln P_t$. This implies that

$$p_t = \eta r - a + \frac{1}{\eta}\int_t^\infty e^{-(s-t)/\eta}\ln(M_s/Y_s)ds, \tag{2.3}$$

where we have assumed that $\lim_{t\to\infty} e^{-t/\eta}\ln P_t = 0$.[17]

2.2. Allowing for Output Effects

As we mentioned in section 1, debt service savings directed to increased spending on poverty reduction may have a significant impact on the government budget through their effect on output. Let the additional government spending on poverty reduction under the initiative

[15] In future work we will extend our analysis to a general equilibrium framework to show that our findings regarding inflation are robust.

[16] This is a standard assumption in any one good model where the price level and the exchange rate must be continuous to avoid predictable arbitrage opportunities.

[17] To see that the solution to (2.2) is indeed given by (2.3), see Appendix B.

be given by $\hat{G}_t$. A simple way to allow for output effects is to assume that some fraction $0 < \alpha \leq 1$ of this spending has an investment component, where investment is defined broadly. We denote the stock of capital specifically built by this investment as K_t. Initially we have $K_0 = 0$. We let K_t evolve according to

$$\dot{K}_t = \alpha \hat{G}_t - \delta K_t \text{ for } t \geq 0. \tag{2.4}$$

To keep our model as simple as possible, we will assume that the level of output, absent debt relief, is some constant, Y, and that the level of output with debt relief is $Y_t = Y + \rho K_t$ for some $\rho \geq 0$. We will assume that tax revenue is proportional to Y_t, i.e. $\Omega_t = \omega Y_t$ for all t.

2.3. An Initial Steady State

We will assume that at time 0 the economy is initially in a steady state where $Y_t = Y$, $G_t = G$, $\Omega_t = \omega Y$, $A_t = A$, $\pi_t = \pi$, $M_t/P_t = m = e^{a-\eta(r+\pi)}Y$. These assumptions imply that

$$\dot{D}_t = rD_t + G - \omega Y - A - \pi m.$$

We also assume that the stock of debt in the initial steady state is constant, so that $\dot{D}_t = 0$, and

$$\pi m + \omega Y = rD_0 + G - A. \tag{2.5}$$

This steady state version of the government budget constraint illustrates, once again, our interpretation of a heavily indebted government. Holding G and A fixed, the higher the level of the government's debt, D_0, the higher its inflation rate must be (over the range $\pi < 1/\eta$, πm is increasing in π), or the more punitive its taxes must be.

We should point out that our analysis does not depend in any crucial way on the initial steady state assumption. Rather, this assumption lends us some analytical convenience without affecting our basic message.

2.4. Characterizing the HIPC Initiative

We interpret the HIPC initiative as an attempt to improve the fiscal position of the recipient government relative to the initial steady state position. In terms of the analysis in section 1 we will work with the first of our three interpretations of the HIPC initiative and its conditionality, though we reiterate that our conclusions are not sensitive to that choice which is merely notational. We first imagine that at time 0, an economy is in a steady state as described by the previous subsection. We then assume that the country receives a previously unanticipated injection of aid that lasts until period T–this captures the finite lifetime of the initiative, though in reality the completion date is often uncertain. In particular we assume that

$$A_t = \begin{cases} A(1+\psi) & \text{for } 0 \leq t < T \\ A & \text{for } t \geq T, \end{cases} \tag{2.6}$$

where $\psi > 0$ determines the increase in aid.[18] We interpret the conditionality of the HIPC initiative as requiring that government expenditure increase by as much as the aid flow until date T.[19] I.e. we assume that

$$G_t = \begin{cases} G + A\psi & \text{for } 0 \leq t < T \\ G & \text{for } t \geq T. \end{cases} \tag{2.7}$$

Given our notation, above, this means $\hat{G}_t = A\psi$ for $0 \leq t < T$ and $\hat{G}_t = 0$ for $t \geq T$.

Since the increase in A_t, in our model, plays the role of decreased debt service payments in the budget constraint, we assume that debtors set ψ in order achieve a particular debt reduction target. In particular, notice that the present value of the debt relief implied by (2.6) is

$$\int_0^\infty (A_t - A)e^{-rt}dt = \int_0^T (A\psi)e^{-rt}dt = A\psi\frac{1-e^{-rT}}{r} \tag{2.8}$$

If θ represents the fraction of a government's debt service that is effectively forgiven, then

$$\theta = A\psi\frac{1-e^{-rT}}{rD_0}. \tag{2.9}$$

2.5. The Path of Output

Our assumptions so far are sufficient to determine the paths of output, Y_t, and tax revenues, Ω_t. We have assumed that $\dot{K}_t = \alpha\hat{G}_t - \delta K_t$ for $t \geq 0$, $K_0 = 0$, $\hat{G}_t = \psi A$ for $0 \leq t \leq T$, $\hat{G}_t = 0$ for $t > T$, and $Y_t = Y + \rho K_t$. With these assumptions we can easily solve for the path of output with debt relief. For $0 \leq t \leq T$, we have $K_t = \int_0^t \alpha(\psi A)e^{\delta(s-t)}ds = \left(1-e^{-\delta t}\right)(\alpha/\delta)\psi A$. For $t > T$ we have $K_t = K_T e^{-\delta(t-T)} = e^{-\delta t}\left(e^{\delta T}-1\right)(\alpha/\delta)\psi A$. Of

[18] Notice that after period T aid flows revert to their former levels. This ignores the possibility, pointed to by Cohen (1999), that donors will decrease their non-debt relief aid once the HIPC initiative is well under way.

[19] Alternatively we could assume that transfers from the government to the private sector increase over the implementation period of the initiative. The equivalence of government purchases and transfers would not carry over to a general equilibrium model.

course, this implies that for $0 \leq t \leq T$, we have $Y_t = Y + \left(1 - e^{-\delta t}\right)(\rho\alpha/\delta)\psi A$ and for $t > T$ we have $Y_t = Y + e^{-\delta t}\left(e^{\delta T} - 1\right)(\rho\alpha/\delta)\psi A$.[20] The path of tax revenue is given by $\Omega_t = \omega Y_t$.

We are now able to measure the increase in the present value of tax revenue due to the effect of the debt relief initiative on output. This is given by

$$\begin{aligned}
\int_0^\infty \omega(Y_t - Y)e^{-rt}dt &= \omega\frac{\rho\alpha}{\delta}\psi A\left[\int_0^T \left(1 - e^{-\delta t}\right)e^{-rt}dt + \int_T^\infty e^{-\delta t}\left(e^{\delta T} - 1\right)e^{-rt}dt\right] \\
&= \omega\rho\alpha\frac{1 - e^{-rT}}{(\delta + r)\,r}\psi A = \frac{\omega\rho\alpha}{\delta + r}\theta D_0. \qquad (2.10)
\end{aligned}$$

Notice that (2.10) relates the increase in the present value of tax revenues to the magnitude of the debt relief package.

2.6. Monetary Policy

To close the model, and describe the paths of prices, inflation and debt, under the initiative, we need to describe monetary policy after date 0. Given that we have determined the paths of A_t, G_t and Ω_t, there are infinitely many paths of the money supply that are consistent with the lifetime budget constraint, (1.7).

To illustrate the effects of different monetary policies, we assume that

$$M_t/Y_t = \begin{cases} (M_0/Y_0)e^{\gamma t} & \text{for } 0 \leq t < T \\ (M_T/Y_T)e^{\bar{\gamma}(t-T)} & \text{for } t \geq T. \end{cases} \qquad (2.11)$$

So we have restricted ourselves to the class of monetary policies in which money grows at a constant rate relative to real GDP, over each of the two subintervals. Within this class of policies there are still infinitely many that satisfy the government's lifetime budget constraint, but we focus on two interesting cases.

In what follows it will be useful to have an expression for the equilibrium inflation rate given that monetary policy is as described in (2.11). In the appendix we establish that (2.3) implies

$$\pi_t = \dot{p}_t = \begin{cases} \gamma + (\bar{\gamma} - \gamma)e^{(t-T)/\eta}. & \text{for } 0 \leq t < T \\ \bar{\gamma} & \text{for } t \geq T, \end{cases} \qquad (2.12)$$

given (2.11).

[20]Note that our model implies that $\lim_{t\to\infty} Y_t = Y$. This means, by assumption, that output eventually returns to its initial steady state value. So our framework is not one in which the investment implicit in the HIPC initiative leads to a permanent change in the growth rate of the economy. We comment on the importance of this assumption in Section 4.

Active Monetary Policy

In our first example, we call monetary policy *active*, because the monetary authority tries to keep inflation stable across the two intervals: $0 \leq t \leq T$ and $t > T$. I.e. the monetary authority chooses γ and $\bar{\gamma}$ so that $\pi_t = \bar{\pi}$, for all $t \geq 0$, for some constant $\bar{\pi}$. From (2.12) it is clear that this requires $\pi_t = \bar{\pi} = \gamma = \bar{\gamma}$ for $t \geq 0$. In doing this, the monetary authority must ensure that the government's lifetime budget constraint, (1.2), is satisfied. Notice that (1.2) can be rewritten as

$$\int_0^\infty \omega(Y_t - Y)e^{-rt}dt = \int_0^\infty (\pi m - \dot{M}_t/P_t)e^{-rt}dt. \tag{2.13}$$

We have already determined the left-hand side of (2.13). We also have π and m. Given our other results, the path of $\dot{M}_t/P_t$ is completely determined by γ. Because (2.13) is a nonlinear equation in γ, we solve it numerically as described in the appendix.

Passive Monetary Policy

We label our second example *passive* monetary policy. In what follows we describe the mechanics of the example and explain why the labeling is natural. We have assumed that government spending increases from G to $G_t = G + \psi A$ in the period $0 \leq t \leq T$. When government spending increases there is a natural tendency for additional liquidity to be injected into the economy. Of course, in our example, the government is also cutting spending—on debt service. But debt service payments are made in foreign currency. So a natural experiment is one in which the government either accumulates reserves, with the savings from debt service, or accumulates new debt less quickly. At the same time the government allows the additional liquidity to finance its increased spending. Offsetting this increase in liquidity is any increase in taxes. Whatever new liquidity is not removed in the form of new taxes represents new seigniorage revenue. The reason we call this passive monetary policy is that the government will almost naturally find itself in this position if it increases domestic spending while reaping savings on the foreign exchange part of its balance sheet.

In terms of our notation this means that the present value of new seigniorage revenue raised between periods 0 and T is given by

$$\int_0^T (\dot{M}_t/P_t - \pi m)e^{-rt}dt = \int_0^T (G_t - G)e^{-rt}dt - \int_0^T \omega(Y_t - Y)e^{-rt}dt. \tag{2.14}$$

That is, new seigniorage through date T is equal to the difference between the present value of new spending and the present value of new taxes through date T. The right-hand side of (2.14) is determined completely by our previous results. The left-hand side of (2.14) is a nonlinear function of γ and $\bar{\gamma}$, so (2.14) represents one equation in our two unknowns, γ and $\bar{\gamma}$.

An interesting implication of passive monetary policy is that it creates a link between the amount of debt the government is left with at date T, and the amount of debt relief it receives under the initiative. To see this, we can roll the budget constraint, (1.1), forward from period 0 to period T to obtain

$$D_0 = e^{-rT} D_T + \int_0^T (\omega Y_t - G_t + A_t + \dot{M}_t / P_t) e^{-rt} dt. \tag{2.15}$$

If we combine (2.14) and (2.15), and use the steady state condition (2.5), we obtain

$$D_T - D_0 = -e^{rT} \int_0^T (A_t - A) e^{-rt} dt = -e^{rT} \theta D_0 \tag{2.16}$$

In other words, by the completion date of the initiative, the government's debt has been reduced by the capitalized value of the debt relief.

It is from this last result that we get our second equation to determine γ and $\bar{\gamma}$. Consider the government's lifetime budget constraint at time T:

$$D_T = \int_T^\infty (\Omega_t - G_t + A_t + \dot{M}_t / P_t) e^{-r(t-T)} dt. \tag{2.17}$$

Given (2.16) and (2.5), (2.17) can be rewritten as

$$-D_0 \theta = \int_T^\infty \omega (Y_t - Y) e^{-rt} dt + \int_T^\infty (\dot{M}_t / P_t - \pi m) e^{-rt} dt. \tag{2.18}$$

This equation states that the present value (at date 0) of taxes and seigniorage raised after date T can decline by the value of the debt relief package. The left-hand side and the first term on the right-hand side of (2.18) are determined by our previous results. The seigniorage term in (2.18) is determined by $\bar{\gamma}$.

So our strategy for solving the model under passive monetary policy is to solve (2.18) numerically for $\bar{\gamma}$. Then we solve (2.14) numerically for γ. This procedure is described in more detail in the appendix.

In the next section of the paper we turn to a quantitative analysis of the model in which we compare the effects of the two monetary policies on the equilibrium paths of prices, inflation and debt. We show how the two policies, active and passive, offer a distinct choice to the policy maker. We should be clear, however, that the point we made in section 1 pertains to all policy choices: given the conditionality of the initiative, the present value of *total* revenues that the government must raise from seigniorage and taxes will be unchanged relative to the initial steady state. If debt relief is effective in raising output, however, raising these revenues may be easier, in the sense that the government may be able to obtain the same revenue with lower tax rates and/or lower inflation. Here we have described a method for solving the model where the tax rate ω is held fixed. Thus, the indirect effects of debt relief, in our examples, all work through an increase in tax revenues, and an offsetting decline in seigniorage revenues.

3. Simulating the Model

In this section of the paper we explore a calibrated example of the model. Using this example we will explore the properties of the equilibrium under the two policy regimes described above. Our first step is to calibrate some of our parameters.

3.1. Calibration

We set the real interest rate, $r = 0.05$. We set the interest elasticity of money demand to $\eta = 0.5$. This value seems broadly consistent with the estimates of interest elasticities reported by Easterly, Mauro, and Schmidt-Hebbel (1995) for developing countries. While our quantitative findings are sensitive to this choice, our qualitative findings are invariant to it. We normalize output as $Y = 1$. We set a, the constant in the Cagan money demand function, to be a value consistent with real balances being 25 percent of GDP in a zero inflation economy. Notice that our model predicts that in such an economy $M_t/(P_tY_t) = e^{a-\eta r}$, so that we set $a = \eta r + \ln(0.25) \approx -1.36$.

We set the initial level of government debt at $D = 0.7Y$, or 70 percent of GDP. We let steady state government purchases $G = 0.2Y$. We assume that the government runs a primary deficit in the absence of aid inflows. That is, we set $\omega = 0.15$, so that in the initial steady state $\Omega = \omega Y < G$. We assume that in the initial steady state, the government receives an aid inflow of 3 percent of GDP. I.e. $A = 0.03Y$.

Our assumptions regarding the initial values of D, G, Ω, and A determine the initial steady state inflation rate and the level of real balances. We have set these values to be such that the government's initial fiscal position is relatively weak—it requires a high inflation rate, and considerable amounts of seigniorage to close its budget constraint. In particular, given our parameter values, steady state seigniorage is given by

$$m\pi = rD + G - \Omega - A = 0.055Y$$

or 5.5 percent of GDP. The steady state inflation rate is about 25 percent. Steady state real balances are $m \approx 0.22Y$.[21]

We assume that the HIPC initiative increases aid to our fictitious country over a 10 year period, i.e. $T = 10$. We set $\psi \approx 0.9$, so that the capitalized value of the forgiven debt service is equal to 50 percent of the country's initial stock of debt, i.e. we set $A\psi(e^{rT} - 1)/r = 0.5D_0 = 0.35Y$. Our assumptions about ψ means that the country receives a flow of debt relief of about 2.7 percent of GDP over the life of the initiative.

In choosing the parameters of the production technology described earlier, we seek a reasonable upper bound for the indirect effects of debt relief with HIPC conditionality. We set δ, the depreciation rate, equal to 0.1. We make relatively generous assumptions about the productivity of aid in generating additional output.[22] We assume that all aid is invested, i.e. $\alpha = 1$, and that the output-capital ratio is 0.5, i.e. $\rho = 0.5$. These seem like generous assumptions to us for the following reason. Suppose we find the value of λ such that

$$\int_0^\infty [Y_t - Y - (G_t - G)]e^{-\lambda t}dt = 0.$$

Clearly λ is the rate of return on the increased social spending.[23] Given our results from above we are looking for λ such that

$$\int_0^T [\left(1 - e^{-\delta t}\right) \frac{\rho\alpha}{\delta}\psi A - \psi A]e^{-\lambda t}dt + \int_T^\infty e^{-\delta t}(e^{\delta T} - 1)\frac{\rho\alpha}{\delta}\psi A e^{-\lambda t}dt = 0.$$

It is straightforward to show that the solution to this equation is

$$\lambda = \rho\alpha - \delta.$$

So, our benchmark example is one in which $\lambda = 0.4$, i.e. the rate of return on social spending is 40 percent. We think this is likely an upper bound for what is plausible.

[21] Having the real monetary base this large is probably counterfactual. However, it is not critical to our qualitative results. In fact, the smaller the monetary base in the steady state, the sharper our results would be: passive monetary policy would be more destabilizing.

[22] For results concerning the effectiveness of foreign aid, see Boone (1996) and Burnside and Dollar (2000).

[23] It is the discount rate that renders the present value of the flow of investment equal to the present value of the flow of payoffs.

3.2. The Lifetime Budget Constraint: Quantitative Results

In Section 1 we emphasized a result concerning the lifetime budget constraint, that was summarized by equation (1.6). This equation states that, with debt relief, the sum of lifetime seigniorage plus tax revenue will be the same as it would have been in the absence of debt relief. Because we have assumed that the tax rate, ω, is unchanged after debt relief, this means that

$$\int_0^\infty (\dot{M}_t/P_t - \pi m)e^{-rt}dt = -\int_0^\infty \omega(Y_t - Y)e^{-rt}dt \tag{3.1}$$

The government can reduce the seigniorage it collects by the amount of its increased tax receipts. We think of this reduction in seigniorage as the extent to which the government budget constraint is indirectly relaxed by debt relief. Using (2.10) we can see that (3.1) can be rewritten

$$\int_0^\infty (\dot{M}_t/P_t - \pi m)e^{-rt}dt = -\frac{\omega\rho\alpha}{\delta + r}\theta D_0. \tag{3.2}$$

In our calibrated examples this means the reduction in lifetime seigniorage is about 11 percent of GDP. This represents about 15 percent of the country's initial stock of debt, about 10 percent of lifetime seigniorage in the steady state, or about 3.5 percent of lifetime tax revenue. In this sense, the indirect effects of debt relief on the government's lifetime budget constraint are relatively modest.

3.3. Active Monetary Policy

In Section 2 we described an active monetary policy under which the government acts to maintain a constant inflation rate for $t \geq 0$. Figure 1 illustrates simulated paths of inflation, the money growth rate, real balances, government debt, foreign aid and seigniorage given the parameter values we selected above. Notice that, in the absence of the growth effect, the economy would never move away from its initial steady state, as indicated by the dashed lines in the figure.

The interesting dynamics in Figure 1 are generated by the growth effect. As we mentioned above, lifetime tax revenue rises by about 11 percent of initial GDP. As a result, lifetime seigniorage can decline by about the same amount. This is reflected in the slightly lower inflation rate after debt relief: about 21 percent, versus 25 percent in the initial steady state. The decline in inflation is small because the increase in lifetime tax revenue only represents about 10 percent of steady state lifetime seigniorage.

In the long-run the stock of debt is permanently reduced by the indirect effects of the debt relief initiative. How much is debt reduced in the long-run? To answer this question we note that in the very long-run, output returns to its steady state level, Y, so that for large t, the flow budget constraint is approximately

$$\dot{D}_t = rD_t + G - \omega Y - A - \gamma \bar{m}$$

where $\bar{m} = e^{a-\eta(r+\bar{\gamma})}Y$. Thus, the stock of debt the economy converges to in the limit is

$$D_A = \frac{\omega Y - G + A + \bar{\gamma}\bar{m}}{r} = D_0 + \frac{\bar{\gamma}\bar{m} - \pi m}{r}.$$

Given our parameter values $D_A \approx 0.55Y$, as compared to the initial stock of debt $D_0 \approx 0.7Y$.

3.4. Passive Monetary Policy

Figure 2 illustrates simulated paths under passive monetary policy. Again, we show simulated paths with and without the indirect growth effect. Both paths indicate that there is a temporary rise in inflation during the period of increased aid and government spending on debt relief. As we stated above, the rise in inflation occurs because there is an instantaneous increase in liquidity that accompanies the increase in government spending. When there is no growth effect, all of this additional liquidity stays in the system and is reflected in higher seigniorage revenue and inflation. When there is a growth effect, some of the additional liquidity leaves the system in the form of increasing tax payments, so the short-run inflation effect is weaker. In our numerical examples, the inflation rate jumps from about 25 percent in the steady state to about 40 percent, if we ignore the growth effect. With the growth effect, inflation only rises to 33 percent in the short-run.

Of course, in the long-run the effect on inflation is reversed. This occurs because the short-run increase in seigniorage revenue leads to a rapid decline in debt. In fact, as we saw above, passive monetary policy naturally leads to a halving of the debt stock by time T. This means that once government spending goes back to its previous levels, the amount of seigniorage needed to close the government budget constraint is greatly reduced. If we ignore the growth effect, the long-run inflation rate is about 16 percent. With the growth effect, the long-run inflation rate is only 14 percent. Again, the growth effect reduces the inflation rate because the increase in tax revenue allows seigniorage revenue to be lower.

How much does debt decline in the long-run with passive monetary policy? For the same reasons given in the previous section, the long-run level of government debt is given by

$$D_P = D_0 + \frac{\bar{\gamma}\bar{m} - \pi m}{r}.$$

When there are no growth effects, $D_P = 0.35Y$. Debt if halved and the government reaches its new steady state level of indebtedness at time T. On the other hand, growth effects contribute to a further reduction in debt, to $D_P = 0.26Y$.[24]

3.5. Summary and Mitigating Factors

From these simulations we take away the following message: debt relief that comes with conditionality requiring increased government spending is likely to have one of two consequences (or some mixture of the two). Either

1. debt is substantially reduced in the long-run, but at the cost of significantly higher inflation during the period of increased spending,[25] and with the benefit of lower inflation later, or

2. through active monetary policy intervention, inflation is kept stable, but there are only modest long-run reductions in government debt and inflation.[26]

Our analysis suggests that there is no easy route to long-run fiscal sustainability via the HIPC initiative as it is narrowly defined here. But our analysis points to two important mitigating factors that can enhance long-run fiscal sustainability. First, it is important that increased spending on poverty reduction be as effective as possible in raising growth in the recipient countries. In our simulations, growth led to an increase in tax revenues that reduced the long-run level of debt by between 9 and 15 percent of GDP, depending on the monetary policy used by the government. But this effect on fiscal sustainability via tax revenues would disappear in the absence of the growth effect.

Second, and more importantly, our analysis points to the importance of reforms not directly linked to HIPC conditionality. In particular, our analysis suggests that HIPC countries

[24] These values of D_P depend on the equilibrium values of $\bar{\gamma}$ and $\bar{m}$ which we compute numerically as described in Section 2.

[25] We experimented with alternative parameter values to see if we could eliminate the short-run rise in the inflation rate in the example with passive monetary policy. We found that in order to do so, we would need to assume that aid had a 110 percent rate of return (almost 3 times the rate of return we assumed in our baseline example).

[26] Our results are the mirror image of Sargent and Wallace's (1981) *unpleasant monetarist arithmetic*: without a change in the primary surplus, low inflation now means more inflation later. In our case, lower inflation in the future means higher inflation now.

need to undertake significant fiscal reforms under which they cut other forms of spending as they raise spending on poverty reduction. Returning to our numerical example, suppose that at time 0 the HIPC government implemented spending cuts symmetric to its increased spending on poverty reduction programs. How would this change our conclusions about the impact of debt relief on inflation and the stock of debt? The answer is straightforward: since the spending cuts would finance the increased spending on poverty reduction, there would be no net injection of liquidity into the economy. Figure 3 illustrates simulations of this scenario with either passive or active monetary policy.[27] With active monetary policy there is a sharp and permanent drop in inflation and debt is substantially reduced by the completion point (date $T = 10$). Even with passive monetary policy there would be a sizeable short-run decline in inflation, due to increased tax receipts, followed by a further decline in inflation after time T. Notice that the monetary policy issues we highlighted previously are now much less important: the paths of inflation are not dramatically different across the two scenarios. Furthermore, the long-run paths of debt are very similar. And, finally, the government's lifetime budget constraint is significantly relaxed. It can finance itself with much less (33 percent) seigniorage revenue.

Some HIPC countries have already undertaken steps towards the types of reforms we have indicated here. That is, they have not only increased spending on poverty reduction, but they have done this in the context of an overall fiscal reform. Our analysis suggests that they are the ones most likely to successfully achieve long-run fiscal sustainability.

4. Shortcomings, Caveats and other FAQs

In this section we highlight some shortcomings of our analysis. We also mention several caveats to it. We also address the relationship between our analysis and other work on debt relief.

1. *Why Doesn't Inflation Have any Impact on Output?*

The main shortcoming we perceive in our analysis is the disconnect between the real and monetary sides of the economy. We determine inflation within the context of a monetary model, but output is an entirely exogenous process with respect to that part of the model. As a result, there is no possibility of feedback from inflation to real activity. Obviously, having

[27] In this example we imagine that $G_t = G$ for all t (because one category of spending rises while another falls). The path of A_t is as before. We assume that output rises as in our other examples, because the government is substituting growth-enhancing expenditure for consumption expenditure.

this sharp dichotomy has its advantages. It makes the model tractable, and allows us to fairly straightforwardly solve for the equilibrium paths of the variables we are interested in. On the other hand, it does not allow us to assess the potentially harmful effects of increasing inflation on output.

2. *Shouldn't the Real Exchange Rate and the Current Account be Affected?*

Again, our model is too simple to capture these effects. We do not model the behavior of private agents other than to specify their demand for money. We also do not differentiate between traded and nontraded goods. As a result, our model does not provide any insight into the behavior of the current account, or the real exchange rate. How these variables behave as a result of debt relief could be another important policy issue for recipient governments.

3. *What if Most of the Increase in Spending is in the Forms of Imports?*

We have characterized the increase in spending for poverty reduction as an increase in government purchases of domestic goods and services or transfers to local residents. As a result, we have assumed that there is an increase in domestic liquidity that the government can respond to passively or actively according to the monetary policies we described. However, to the extent that increased spending on poverty reduction is in the form of imports, the differences between the two monetary policies will be less stark. Passive monetary policy will start to look more like active monetary policy, because the increased expenditures on imports will not inject liquidity into the domestic economy.

4. *Does the Fact that the Initial Condition is a Steady State Matter?*

Yes and no. This matters to the extent that our simulations compare the paths of inflation, debt, and other variables to the pre debt-relief steady state. It does not matter for the paths themselves. Take, as an example, the path of inflation under passive monetary policy. The path shown in Figure 2 shows a short-run rise in inflation relative to the initial steady state, followed by a decline in inflation relative to the steady state. If we did not assume the economy was in a steady state prior to date 0, the only thing that would change in Figure 2 would be the interval prior to date 0.

5. *Would the Results Change if Long-Run Growth Was Built Into the Model?*

Wehave used a model with no steady state growth for simplicity. This means that after the debt-relief initiative is concluded, the economy eventually comes back to a position where output is at the constant level Y. Since our approach involves comparing equilibrium paths with debt relief, to equilibrium paths without debt relief, the *comparisons* would be the same if we built a common long-run real growth rate into the two scenarios. This is because the comparisons made in our analysis center on the once-and-for-all characteristic of the debt relief initiative. We would have to assume that the *long-run* real growth rate was different across the two scenarios for our results to change qualitatively.

6. *For which countries would the message on inflation be most relevant?*

Given our simulations, we think there are three types of countries for which our message about inflation is most relevant. First, we have seen that the alternative monetary policies lead to starkly different outcomes in the simulations where the recipient country has not undertaken significant fiscal reforms that would independently finance increased spending on poverty reduction. So one set of countries for which our message is most relevant are those countries that have not yet reached the decision point for debt relief under the initiative. Second are those countries starting from a high initial inflation level who will presumably find it most difficult to keep inflation under control and meet the conditions on macroeconomic stability agreed upon with donors. Third are those countries for whom the increased spending on poverty reduction represents the largest percentage increases in their expenditure envelopes.

7. *Is the Fact that Seigniorage is High in the Steady State a Key Assumption?*

For our quantitative results every parameter choice matters to some degree. Our way of representing an initial lack of fiscal sustainability is to assume that the government requires a great deal of seigniorage to close its budget constraint. Our way of measuring the relaxation of its budget constraint is to see how much its reliance on seigniorage is reduced by debt relief. Alternatively, we could have assumed that the government initially closes its budget constraint through punitive distortionary taxation with very little or no seigniorage. We could then measure the relaxation of its budget constraint by the extent to which it could lower tax rates after receiving debt relief. Or, finally, we could assume that neither taxes nor seigniorage was very high, and instead assume that the government was initially reliant on the accumulation of arrears. We would then measure the relaxation of the government budget constraint by the extent to which the accumulation of arrears declined with debt-relief. Our focus on seigniorage is simply one way of talking about the issue that quantifies it. If we focused on one of the two alternatives mentioned here, we would draw similar conclusions from the lifetime budget constraint: debt relief with HIPC conditionality would provide a modest relaxation of the budget constraint. Our other points would also be robust:

(i) under passive monetary policy there would be a short-run rise in inflation followed by a long-run decline, and (ii) ensuring that poverty reduction spending was growth enhancing and introducing significant fiscal reforms would be key measures needed to ensure a longer run improvement in the government's fiscal position together with macroeconomic stability.

5. Concluding Remarks

In this paper, we have discussed the potential macroeconomic impact of the HIPC initiative. Our analysis suggests that short-run inflationary pressure might arise as a result of increased domestic spending for poverty reduction. Some recipient countries may be ill-equipped to deal with these pressures and could experience a substantial temporary increase in inflation. While our model does not capture the effect, this increase in inflation could impact negatively on growth. Our model suggests that recipient countries can use monetary policy to stabilize inflation, but, in doing so, they will tend to limit the longer run impact of the HIPC initiative on their debt levels.

We have also shown that the impact on inflation will be lower if increased government spending produces favorable output effects. Therefore the analysis suggests that in order to minimize the extent of macroeconomic destabilization, care should be taken to eliminate structural bottlenecks to growth.

The concerns we have raised will be less relevant when the recipient country has already achieved significant progress in macroeconomic stabilization. In those cases where the recipient government has already implemented fiscal reforms that would enable it to reduce its debt level over time, the risk of short-run inflation will be diminished and longer run fiscal sustainability will be ensured.

A major shortcoming of our analysis is that our simple analytical framework does not allow us to explore more fully the impact of debt relief on real activity, the real exchange rate, and the external current account. In future research, we intend to extend our analysis to deal with these issues, using a general equilibrium small-open-economy model.

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Appendices

A. Basic Accounting Examples

In our basic accounting examples we imagine a government with a zero operational surplus, which, by assumption, *must* amortize X units of the public sector's net nonmonetary debt by the end of the period. In our benchmark case, where the government receives no outside aid, we will show that—given the zero operational surplus—the central bank will have to print X units of money (seigniorage) to finance the debt reduction. In our second case, where the government receives additional outside aid, it can reduce its reliance on seigniorage by the amount of the aid received, and still pay off the same amount of debt. This, of course, will assume that the government continues to have a zero operational surplus. Finally, in our third example, we will consider the case where the government receives aid but, as under the HIPC initiative, it commits itself to increased spending on goods and services equal to the value of the aid it receives. Abstracting from the beneficial output effects this spending might have, this we will see that once again the central bank must print X units of money to finance the debt reduction. It is in this sense, that aid with HIPC conditionality leaves the government's fiscal position unchanged relative to the benchmark case.

We will work with simple central bank and government balance sheets as shown in Table 1. Imagine that these balance sheets have some net position at the beginning of the period. We assume that within the period the government spends $G + I$ units (on goods, services and its real interest bill) and raises T units of tax revenue. Notice that when the government purchases the goods and services it will do so, effectively, by drawing a cheque on its deposit account at the central bank. When this cheque clears, the monetary base will have risen by $G + I$ units and government deposits will have declined by $G + I$ units. However, since taxpayers will use their money balances to pay taxes, once the government raises its tax revenue, the T-accounts are restored to their original positions if we assume that $G+I = T$. This leaves only the question as to how the government will finance the debt reduction. Notice that its only option is to print money, whether it pays off foreign debt or domestic debt. It cannot use foreign exchange reserves, because in so doing, it will not change the *public sector's* net liability position. It cannot issue new debt, for the same reason. Hence it must let the monetary base rise by X units, as in Table 1(a), by writing a cheque on its deposit account. If the government chooses to pay off foreign debt, as in Table 1(a), it must acquire foreign assets from domestic residents in order to do so (this assumes foreigners will not hold domestically denominated claims). On the other hand, if the government chooses to pay off domestic debt, the final position is as in Table 1(b).

TABLE 1

AN EXAMPLE WITH NO DEBT RELIEF

(a) Government Pays off Foreign Debt

Central Bank			Government			
Assets	Liabilities		Assets		Liabilities	
N/C	Base Money	$+X$	Deposits @ C.B.	$-X$	Foreign Debt	$-X$
	Gov't Deposits	$-X$				
			Net nonmonetary debt of the public sector:			$-X$

(b) Government Pays off Domestic Debt

Central Bank			Government			
Assets	Liabilities		Assets		Liabilities	
N/C	Base Money	$+X$	Deposits @ C.B.	$-X$	T-Bills	$-X$
	Gov't Deposits	$-X$				
			Net nonmonetary debt of the public sector:			$-X$

N/C indicates no change.

If the government were to obtain aid from a foreign source, as long as it left taxes and spending on goods and services unchanged, it would, of course, be able to achieve its debt reduction without resort to seigniorage. For example, if the aid were literally in the form of X units of foreign debt cancelled, the public sector's financial position would be described by Table 2(a). That table would also describe the result of the government actually receiving aid, if it used the aid to pay off its foreign debt. An alternative for the government, might also be, as in Table 2(b), to simply pay off some of its domestic debt. To achieve this, the central bank would sell its increased foreign exchange reserves (from the aid inflow) to domestic residents in exchange for their holdings of T-bills.[28] Since the T-bills are held by the central bank, they are no longer part of the public sector's net stock of debt. The important thing to notice is that either way, the aid eliminates the government's need to raise seigniorage.

[28] The example assumes that the Central Bank's reserves initially rise by X units as a result of the aid inflow. As a result it credits the government's deposit account X extra units. The central bank then swaps the X units of reserves for X units of the public's holdings of T-bills.

TABLE 2

An Example with Debt Relief, No HIPC Conditionality

(a) Foreign Debt Cancellation

Central Bank		Government		
Assets	Liabilities	Assets	Liabilities	
N/C	N/C	N/C	Foreign Debt	$-X$
			Net nonmonetary debt of the public sector:	$-X$

(b) Domestic Debt Reduction

Central Bank				Government		
Assets		Liabilities		Assets		Liabilities
T-Bills	$+X$	Gov't Deposits	$+X$	Deposits @ C.B.	$+X$	N/C
					Net nonmonetary debt of the public sector:	$-X$

TABLE 3

Examples with Debt Relief and Conditionality

(a) Debt is Reduced

Central Bank			Government			
Assets	Liabilities		Assets		Liabilities	
N/C	Base Money	$+X$	Deposits @ C.B.	$-X$	Foreign Debt	$-X$
	Gov't Deposits	$-X$				
					Net nonmonetary debt of the public sector:	$-X$

(b) Sterilization through FX Market Intervention

Central Bank				Government			
Assets		Liabilities		Assets		Liabilities	
FX Res.	$-X$	Gov't Deposits	$-X$	Deposits @ C.B.	$-X$	Foreign Debt	$-X$
						Net nonmonetary debt of the public sector:	N/C

(c) Sterilization through an Open Market Operation

Central Bank				Government			
Assets		Liabilities		Assets		Liabilities	
T-Bills	$-X$	Gov't Deposits	$-X$	Deposits @ C.B.	$-X$	Foreign Debt	$-X$
						Net nonmonetary debt of the public sector:	N/C

Now suppose that when the government receives debt relief, as in Table 2(a), it also must commit itself to an increase in government purchases of domestic goods and services, or transfers to domestic residents, of X units. If we assume that the government does not raise new taxes to finance this increased spending, then it must, as before, print money. The public sector accounts end up looking like Table 3(a). The effect on the money supply of the government's increased spending can, of course, be sterilized by the central bank. It can do this by selling its foreign exchange reserves (as in Table 3(b)) in the foreign exchange market. It can also sterilize by selling T-bills in an open market operation (as in Table 3(c)). Notice, however, that in both these examples, the government's net debt position remains at its initial position. It does not achieve what we had assumed was a *required* debt reduction.

These simple examples illustrate two of our main arguments outlined in the introduction. First, if we compare the three scenarios described here, we see that the first and third scenarios are ones in which the government's fiscal positions are identical–the fact that new aid flows commit the government to increased spending mean that its fiscal position is unchanged. This is because, in a static context, the government's fiscal position is summarized by its operational surplus inclusive of aid. This is the same across the first and third examples. In the second example, aid is received absent any commitment to increased spending so the government'ss fiscal position improves. The analysis in the body of the text is simply an extension to the dynamic context.

Second, the scenarios considered in Table 3 illustrate the monetary policy dilemma faced by a HIPC country. To obtain the fiscal benefit of debt relief, the government must follow a passive monetary policy. If the central bank actively mops up liquidity, as illustrated in Tables 3(b) or 3(c), this will tend to undo any reduction in the government's indebtedness.

B. Solving the Model

B.1. Two Useful Formulas

Here we derive expressions for $\int_a^b e^{-(s-t)/\eta}ds$ and $\int_a^b se^{-(s-t)/\eta}ds$. First, we note that for any function $e^{\psi x}$

$$\int e^{\psi x}dx = C + e^{\psi x}/\psi \tag{B.1}$$

and

$$\int xe^{\psi x}dx = C + (x - 1/\psi)\left(e^{\psi x}/\psi\right) \tag{B.2}$$

where in each case C is some arbitrary constant of integration. It is straightforward to verify these solutions. Notice that the derivative of the right-hand-side of (B.1) with respect to x is $e^{\psi x}$, while the derivative of the right-hand-side of (B.2) with respect to x is $xe^{\psi x}$.

Hence we can write

$$\begin{aligned}\int_a^b e^{-(s-t)/\eta}ds &= e^{t/\eta}\int_a^b e^{-s/\eta}ds \\ &= e^{t/\eta}(-\eta e^{-s/\eta})|_a^b\end{aligned}$$

$$= \quad -\eta e^{t/\eta}(e^{-b/\eta} - e^{-a/\eta}) \tag{B.3}$$

and

$$\begin{aligned}
\int_a^b s e^{-(s-t)/\eta} ds &= e^{t/\eta} \int_a^b s e^{-s/\eta} ds \\
&= e^{t/\eta}[(s+\eta)(-\eta e^{-s/\eta})]|_a^b \\
&= -\eta e^{t/\eta}[(b+\eta)\, e^{-b/\eta} - (a+\eta)\, e^{-a/\eta}].
\end{aligned} \tag{B.4}$$

B.2. Solving the Cagan Model

The solution to the Cagan model given in (2.3) is

$$p_t = \eta r - a + \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} \ln(M_s/Y_s) ds. \tag{B.5}$$

Differentiating both sides of this equation with respect to t we obtain

$$\dot{p}_t = -\frac{1}{\eta} \ln(M_t/Y_t) + \frac{1}{\eta^2} \int_t^\infty e^{-(s-t)/\eta} \ln(M_s/Y_s) ds. \tag{B.6}$$

We can replace the integral in (B.6) using (B.5) to obtain:

$$\dot{p}_t = -\frac{1}{\eta} \ln(M_t/Y_t) + \frac{1}{\eta}(p_t - \eta r + a).$$

Multiplying this by η we obtain an identical equation to (2.2):

$$\eta \dot{p}_t = \ln Y_t - \ln M_t + p_t - \eta r + a.$$

Given the monetary policy described in (2.11), for $0 \le t \le T$ we have:

$$\begin{aligned}
p_t &= \eta r - a + \frac{1}{\eta} \int_t^T e^{-(s-t)/\eta} \ln[(M_0/Y_0)e^{\gamma s}] ds + \frac{1}{\eta} \int_T^\infty e^{-(s-t)/\eta} \ln[(M_T/Y_T)e^{\bar{\gamma}(s-T)}] ds \\
&= \eta r - a + \frac{\ln(M_0/Y_0)}{\eta} \int_t^T e^{-(s-t)/\eta} ds + \frac{\ln(M_T/Y_T) - \bar{\gamma}T}{\eta} \int_T^\infty e^{-(s-t)/\eta} ds + \\
&\quad \frac{1}{\eta} \int_t^T e^{-(s-t)/\eta} \gamma s ds + \frac{1}{\eta} \int_T^\infty e^{-(s-t)/\eta} \bar{\gamma} s ds \\
&= \eta r - a - \ln(M_0/Y_0)(e^{(t-T)/\eta} - 1) + [\ln(M_T/Y_T) - \bar{\gamma}T] e^{(t-T)/\eta} - \\
&\quad \gamma[(T+\eta)\, e^{(t-T)/\eta} - (t+\eta)] + \bar{\gamma}\,(T+\eta)\, e^{(t-T)/\eta} \\
&= \eta r - a + \ln(M_0/Y_0) + \gamma(t+\eta) - \eta(\gamma - \bar{\gamma}) e^{(t-T)/\eta}.
\end{aligned} \tag{B.7}$$

On the other hand, for $t > T$ we have

$$\begin{aligned}
p_t &= \eta r - a + \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} \ln[(M_T/Y_T)e^{\bar{\gamma}(s-T)}] ds \\
&= \eta r - a + \frac{\ln(M_T/Y_T) - \bar{\gamma}T}{\eta} \int_t^\infty e^{-(s-t)/\eta} ds + \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} \bar{\gamma} s ds \\
&= \eta r - a + \ln(M_0/Y_0) + (\gamma - \bar{\gamma})T + \bar{\gamma}\,(t+\eta)\,.
\end{aligned} \tag{B.8}$$

Hence the inflation rate for $0 \le t \le T$ is

$$\pi_t = \dot{p}_t = \gamma - (\gamma - \bar{\gamma})e^{(t-T)/\eta}$$

while for $t \ge T$ it is simply $\pi_t = \bar{\gamma}$.

B.3. Details of the Analysis in Section 2

Active Monetary Policy

We need to find a γ such that (2.13) is satisfied. From (2.10), we can rewrite (2.13) as

$$\frac{\omega\rho\alpha}{\delta + r}\theta D_0 = \int_0^\infty (\pi m - \dot{M}_t/P_t)e^{-rt}dt. \tag{B.9}$$

Given (2.11), (B.7) and (B.8) we know that $P_t = e^{-a+\eta(r+\gamma)}(M_0/Y_0)e^{\gamma t}$, and that $\dot{M}_t = (M_0/Y_0)e^{\gamma t}\dot{Y}_t + \gamma(M_0/Y_0)e^{\gamma t}Y_t$. Therefore,

$$\frac{\dot{M}_t}{P_t} = e^{a-\eta(r+\gamma)}(\dot{Y}_t + \gamma Y_t) = b[\gamma Y + \dot{Y}_t + \gamma(Y_t - Y)]$$

where $b = e^{a-\eta(r+\gamma)}$. Given our results about the path of output in section 2, we can then rewrite (B.9) as

$$\begin{aligned}\frac{\omega\rho\alpha}{\delta + r}\theta D_0 &= \frac{\pi m}{r} - \frac{\gamma b Y}{r} - b\rho\alpha\psi A\left[\int_0^T e^{-(r+\delta)t}dt + (\gamma/\delta)\int_0^T \left(1 - e^{-\delta t}\right)e^{-rt}dt\right] + \\ & \quad b\rho\alpha\psi A\left(1 - \gamma/\delta\right)\left(e^{\delta T} - 1\right)\int_T^\infty e^{-(r+\delta)t}dt.\end{aligned}$$

This can be rewritten as

$$\frac{\omega\rho\alpha}{\delta + r}\theta D_0 = \frac{1}{r}\left\{\pi m - e^{a-\eta(r+\gamma)}\left[\gamma Y + \rho\alpha\psi A\frac{\left(1 - e^{-rT}\right)(r+\gamma)}{(r+\delta)}\right]\right\} \tag{B.10}$$

We solve (B.10) numerically for γ.

Passive Monetary Policy

In this section we describe how we solve for γ and $\bar{\gamma}$ under passive monetary policy. The first step in our analysis is to solve (2.18) for $\bar{\gamma}$. Given (2.11) and (B.8) we have $P_t = e^{\eta(r+\bar{\gamma})-a}(M_0/Y_0)e^{(\gamma-\bar{\gamma})T+\bar{\gamma}t}$ and $\dot{M}_t = \dot{Y}_t(M_0/Y_0)e^{\gamma T+\bar{\gamma}(t-T)} + \bar{\gamma}Y_t(M_0/Y_0)e^{\gamma T+\bar{\gamma}(t-T)}$ for $t \ge T$. Hence, for $t > T$,

$$\frac{\dot{M}_t}{P_t} = e^{a-\eta(r+\bar{\gamma})}(\dot{Y}_t + \bar{\gamma}Y_t) = \bar{b}[\bar{\gamma}Y + \dot{Y}_t + \bar{\gamma}(Y_t - Y)]$$

where $\bar{b} = e^{a-\eta(r+\bar{\gamma})}$. This allows us to rewrite (2.18) as

$$-D_0\theta = \int_T^\infty \left\{(\omega + \bar{b}\bar{\gamma})(Y_t - Y) + \bar{b}(\bar{\gamma}Y + \dot{Y}_t)\right\} e^{-rt}dt - \int_T^\infty \pi m e^{-rt}dt$$

or, given our results on the path of output in section 2:

$$-D_0\theta = \rho\alpha\psi A\left(1 - e^{-\delta T}\right)\frac{\omega + e^{a-\eta(r+\bar{\gamma})}(\bar{\gamma} - \delta)}{\delta(\delta + r)}e^{-rT} + [\bar{\gamma}e^{a-\eta(r+\bar{\gamma})}Y - \pi m]\frac{1}{r}e^{-rT} \tag{B.11}$$

We solve (B.11) numerically for $\bar{\gamma}$.

Next we solve (2.14) for γ. To do this we note that (2.11) and (B.7) imply $P_t = e^{\eta(r+\gamma)-a}(M_0/Y_0)\exp(\gamma t - \eta(\gamma - \bar{\gamma})e^{(t-T)/\eta})$ and $\dot{M}_t = (M_0/Y_0)e^{\gamma t}\dot{Y}_t + \gamma(M_0/Y_0)e^{\gamma t}Y_t$ for $0 \leq t \leq T$. Therefore, for $0 \leq t \leq T$,

$$\frac{\dot{M}_t}{P_t} = \frac{\gamma Y + \dot{Y}_t + \gamma(Y_t - Y)}{e^{\eta(r+\gamma)-a}\exp(-\eta(\gamma - \bar{\gamma})e^{(t-T)/\eta})}.$$

This allows us to write

$$\begin{aligned}\int_0^T \frac{\dot{M}_t}{P_t}e^{-rt}dt \quad &= \quad \gamma e^{a-\eta(r+\gamma)}\left(Y + \frac{\rho\alpha}{\delta}\psi A\right)\int_0^T \exp[-rt + \eta(\gamma - \bar{\gamma})e^{(t-T)/\eta}]dt + \\ &\left(1 - \frac{\gamma}{\delta}\right)e^{a-\eta(r+\gamma)}\rho\alpha\psi A\int_0^T \exp[-(r+\delta)t + \eta(\gamma - \bar{\gamma})e^{(t-T)/\eta}]dt\end{aligned} \tag{B.12}$$

We can write

$$\int_0^T \pi m e^{-rt}dt = \pi m\frac{1 - e^{-rT}}{r}. \tag{B.13}$$

The expressions on the right-hand side of (2.14) are

$$\int_0^T (G_t - G)e^{-rt}dt = \psi A\frac{1 - e^{-rT}}{r} \tag{B.14}$$

and

$$\int_0^T \omega(Y_t - Y)e^{-rt}dt = \omega\frac{\rho\alpha}{\delta}\psi A\int_0^T\left(1 - e^{-\delta t}\right)e^{-rt}dt = \omega\frac{\rho\alpha}{\delta}\psi A\frac{re^{-T(\delta+r)} - e^{-rT}(r+\delta) + \delta}{(\delta + r)\, r}. \tag{B.15}$$

Using (B.12)–(B.15) we can solve (2.14) numerically for γ given $\bar{\gamma}$.

B.4. Details of the Analysis in Section 3

Simulations in Figures 1 and 2

In the previous subsection of the appendix, we have shown how to obtain the equilibrium values of γ and $\bar{\gamma}$. Given the path of output, γ and $\bar{\gamma}$ completely determine the paths of M_t

and P_t. In some of our simulations we also would like to have expressions for the stock of debt, D_t. We note that for $0 \leq t \leq T$, the stock of debt evolves according to

$$D_t = e^{rt} D_0 - e^{rt} \int_0^t (\omega Y_s - G_s + A_s + \dot{M}_s / S_s) e^{-rs} ds \tag{B.16}$$

We note that (2.5) lets us rewrite this as

$$D_t = D_0 - e^{rt} \left[\int_0^t \omega (Y_s - Y) e^{-rs} ds + \int_0^t (\dot{M}_s / S_s - \pi m) e^{-rs} ds \right]. \tag{B.17}$$

We can write the first integral in (B.17) as

$$\begin{aligned} \int_0^t \omega (Y_s - Y) e^{-rs} ds &= \omega \frac{\rho \alpha}{\delta} \psi A \int_0^t \left(1 - e^{-\delta s}\right) e^{-rs} ds \\ &= \omega \frac{\rho \alpha}{\delta} \psi A \frac{r e^{-t(\delta + r)} - e^{-rt}(r + \delta) + \delta}{(\delta + r) r} \end{aligned} \tag{B.18}$$

Given our results above, we can write the second integral in (B.17) as

$$\begin{aligned} \int_0^t \left(\frac{\dot{M}_s}{P_s} - \pi m\right) e^{-rs} ds &= b \Big\{ \gamma \left(Y + \frac{\rho \alpha}{\delta} \psi A \right) \int_0^t \exp[-rs + \eta(\gamma - \bar{\gamma}) e^{(s-T)/\eta}] ds + \\ &\quad \left(1 - \frac{\gamma}{\delta}\right) \rho \alpha \psi A \int_0^t \exp[-(r + \delta)s + \eta(\gamma - \bar{\gamma}) e^{(s-T)/\eta}] ds \Big\} - \\ &\quad \pi m \frac{1 - e^{-rt}}{r} \end{aligned} \tag{B.19}$$

where $b = e^{a - \eta(r + \gamma)}$. Under active monetary policy this reduces to

$$\int_0^t \left(\frac{\dot{M}_s}{P_s} - \pi m\right) e^{-rs} ds = (\gamma b Y - \pi m) \frac{1 - e^{-rt}}{r} + b \rho \alpha \psi A \left[\frac{\gamma}{\delta} \frac{1 - e^{-rt}}{r} + \left(1 - \frac{\gamma}{\delta}\right) \frac{1 - e^{-(r+\delta)t}}{\delta + r} \right]. \tag{B.20}$$

For $t > T$ we note that (B.17) still holds. The first integral on the right-hand side of (B.17) is

$$\begin{aligned} \int_0^t \omega (Y_s - Y) e^{-rs} ds &= \omega \frac{\rho \alpha}{\delta} \psi A \left[\int_0^T \left(1 - e^{-\delta s}\right) e^{-rs} ds + \left(e^{\delta T} - 1\right) \int_T^t e^{-(r+\delta)s} ds \right] \\ &= \omega \frac{\rho \alpha}{\delta} \psi A \frac{\delta (1 - e^{-rT}) + r e^{-t(\delta + r)} (1 - e^{\delta T})}{(\delta + r) r}. \end{aligned} \tag{B.21}$$

For $t > T$, the second integral on the right-hand side of (B.17) becomes

$$\int_0^t \left(\frac{\dot{M}_s}{P_s} - \pi m\right) e^{-rs} ds = \int_0^T \left(\frac{\dot{M}_s}{P_s} - \pi m\right) e^{-rs} ds + \int_T^t \left(\frac{\dot{M}_s}{P_s} - \pi m\right) e^{-rs} ds. \tag{B.22}$$

The first part of the right-hand side of (B.22) can be evaluated using (B.19) (at $t = T$). The second part is given by

$$\int_T^t \left(\frac{\dot{M}_s}{P_s} - \pi m\right) e^{-rs} ds = \int_T^t \bar{b} [\bar{\gamma} Y + (\bar{\gamma} - \delta) e^{-\delta s} \left(e^{\delta T} - 1\right) \frac{\rho \alpha}{\delta} \psi A] e^{-rs} ds - \pi m \frac{e^{-rT} - e^{-rt}}{r},$$

where $\bar{b} = e^{a-\eta(r+\bar{\gamma})}$, or

$$\begin{aligned}\int_T^t (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds &= (\bar{\gamma}\bar{b}Y - \pi m)\frac{e^{-rT} - e^{-rt}}{r} + \\ &\bar{b}(\bar{\gamma} - \delta)\left(e^{\delta T} - 1\right)\frac{\rho\alpha}{\delta}\psi A \frac{e^{-T(\delta+r)} - e^{-t(\delta+r)}}{r+\delta}. \end{aligned} \tag{B.23}$$

Simulations in Figure 3

In this section we imagine that $G_t = G$, for $t \geq 0$. The paths of A_t and Y_t are the same as in our other examples. We now show how to solve the model under active and passive monetary policy.

Under active monetary policy the government chooses the single parameter of monetary policy, $\gamma = \bar{\gamma}$, to satisfy its lifetime budget constraint, (1.2). With our new assumptions this becomes

$$\int_0^\infty \left[\omega(Y_t - Y) + (A_t - A) + (\dot{M}_t/P_t - \pi m)\right] e^{-rt}dt = 0 \tag{B.24}$$

The first part of this expression is given by (2.10). The second part is given by (2.8). The negative of the third part appears on the right-hand side of (B.10). Combining these terms we have

$$\left(\frac{\omega\rho\alpha}{\delta+r} + 1\right)\frac{1-e^{-rT}}{r}\psi A = \frac{1}{r}\left\{\pi m - e^{a-\eta(r+\gamma)}\left[\gamma Y + \rho\alpha\psi A \frac{\left(1-e^{-rT}\right)(r+\gamma)}{(r+\delta)}\right]\right\}$$

which we solve for γ.

Under passive monetary policy we need to replace (2.14) by

$$\int_0^T (\dot{M}_t/P_t - \pi m)e^{-rt}dt = -\int_0^T \omega(Y_t - Y)e^{-rt}dt \tag{B.25}$$

since there is no natural injection of liquidity. If we combine (B.25) and (2.15), and use the steady state condition (2.5), we again obtain (2.16). We also obtain (2.18) as in Section 2. This immediately implies that the solution for $\bar{\gamma}$ is the same as before. But the solution for γ is different. To solve for γ, using (B.25) we note that the expressions in (B.12), (B.13) and (B.15) can be used.

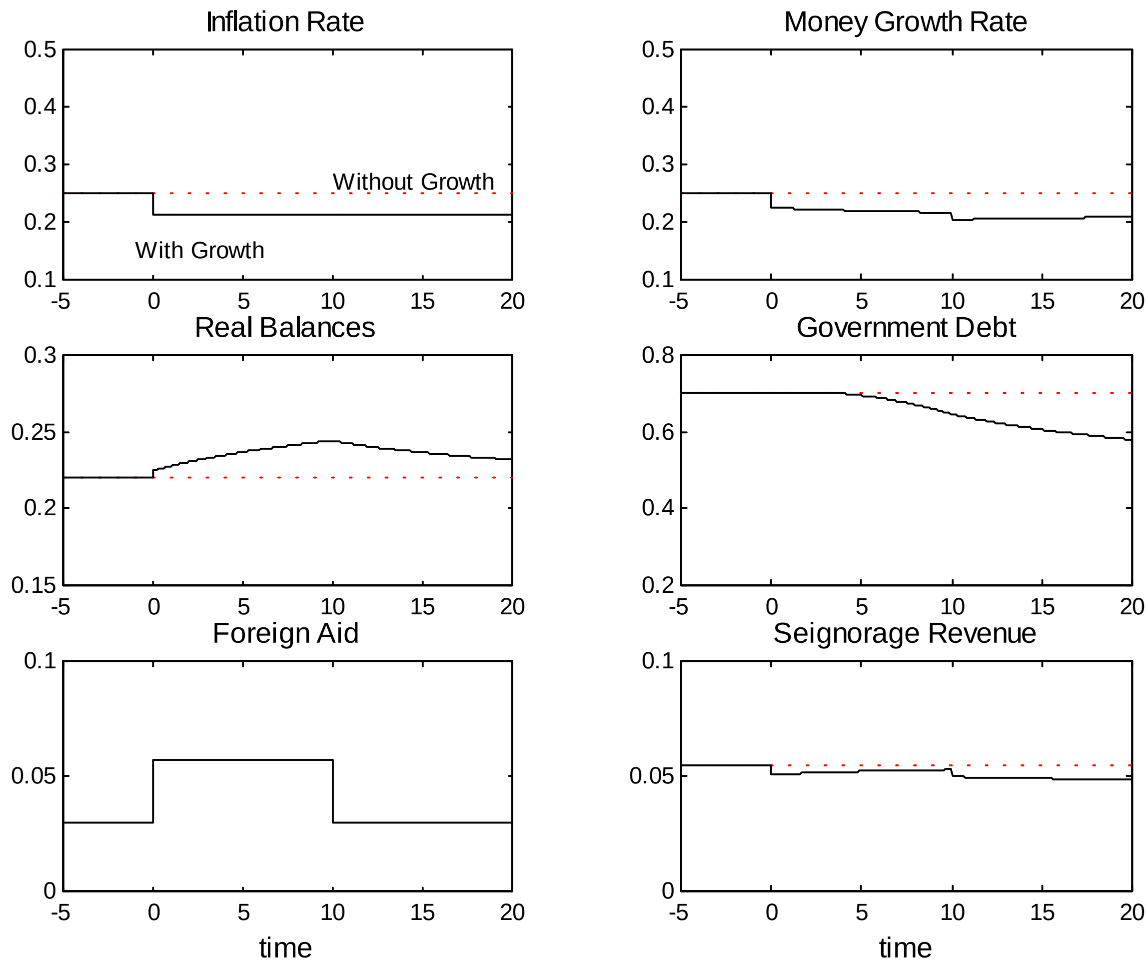
To simulate the path of debt under either scenario we note that the previous formulas apply with one exception. The path of debt for $0 \leq t \leq T$ follows

$$D_t = D_0 - e^{rt}\left[\int_0^t \omega(Y_s - Y)e^{-rs}ds + \int_0^t (A_t - A)e^{-rs}ds + \int_0^t (\dot{M}_s/S_s - \pi m)e^{-rs}ds\right]$$

where $\int_0^t (A_t - A)e^{-rs}ds = \psi A(1 - e^{-rt})/r$.

FIGURE 1

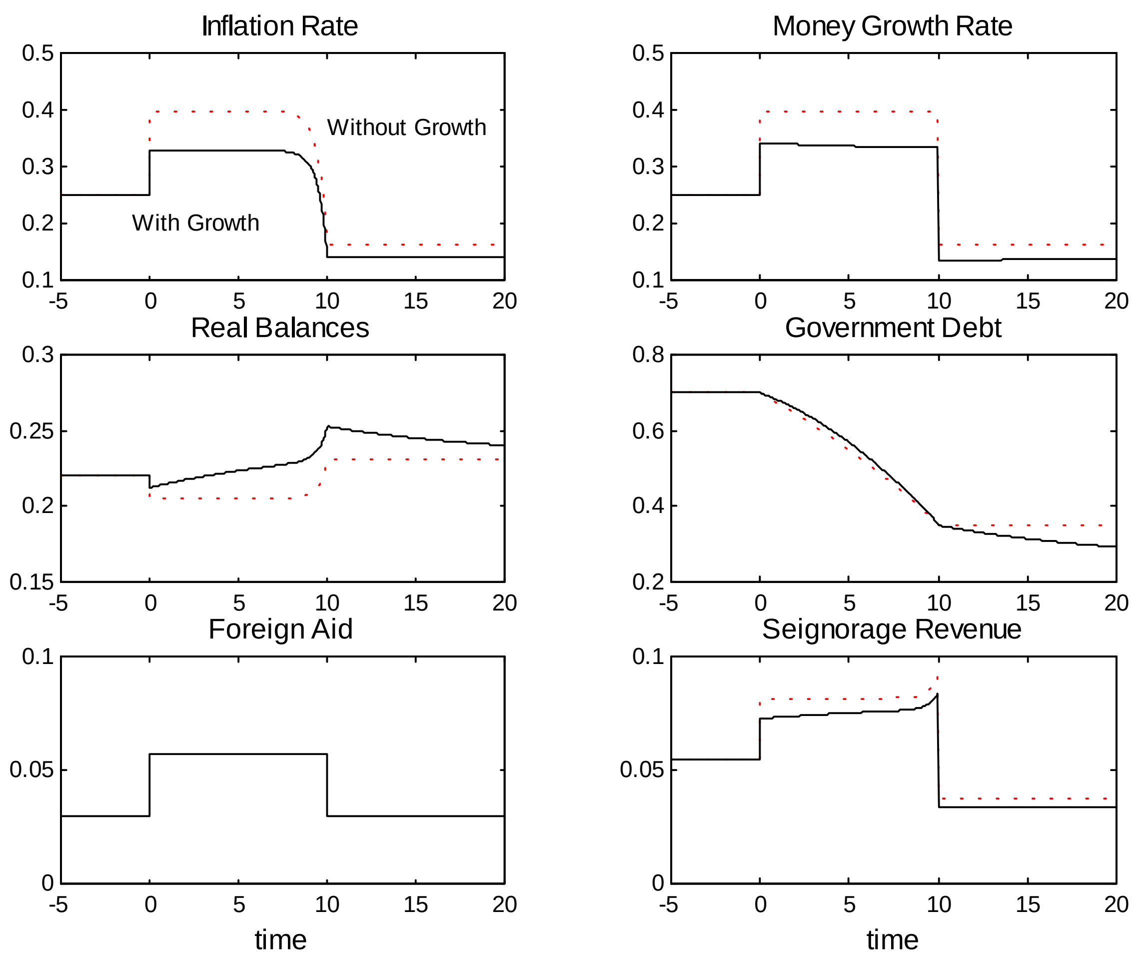
SIMULATIONS UNDER ACTIVE MONETARY POLICY



Notes: In these examples, the government actively smoothes inflation over time. The solid lines indicate the paths of each variable assuming that the increased spending on poverty reduction raises output. The dashed lines indicate what these paths would be in the absence of this additional growth. So the dashed lines indicate the direct effect of debt relief with HIPC conditionality, while the solid lines indicate the combination of the direct and indirect effects. The scales for real balances, debt, aid and seignorage can all be interpreted by noting that GDP = 1. The inflation and money growth rates are expressed in decimal form.

FIGURE 2

SIMULATIONS UNDER PASSIVE MONETARY POLICY

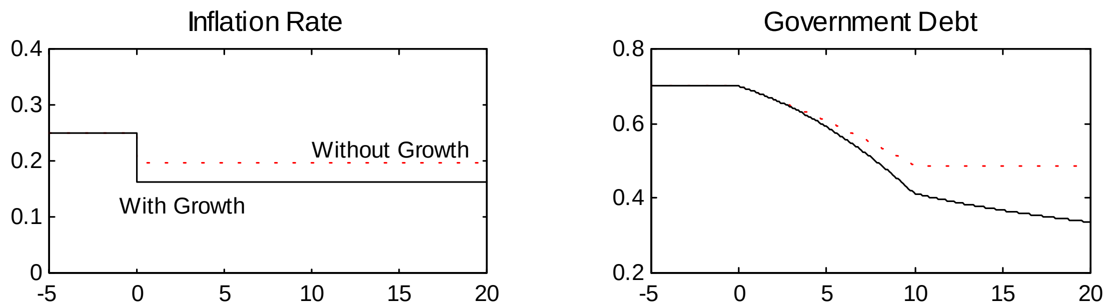


Notes: In these examples, the government does not actively intervene to smooth inflation. It allows the liquidity associated with the increase in government spending to stay in the economy to the extent that increased tax revenues do not remove it. The solid lines indicate the paths of each variable assuming that the increased spending on poverty reduction raises output. The dashed lines indicate what these paths would be in the absence of this additional growth. So the dashed lines indicate the direct effect of debt relief with HIPC conditionality, while the solid lines indicate the combination of the direct and indirect effects. The scales for real balances, debt, aid and seignorage can all be interpreted by noting that GDP = 1. The inflation and money growth rates are expressed in decimal form.

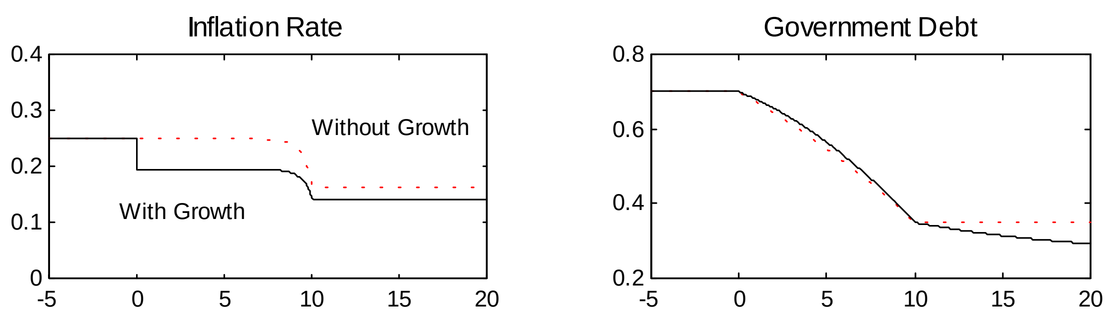
FIGURE 3

SIMULATIONS WITH ADDITIONAL FISCAL REFORMS

(a) Active Monetary Policy



(b) Passive Monetary Policy



Notes: In these examples, the government implements spending cuts symmetric to its spending increase over the life of the debt relief initiative. The solid lines indicate the paths of each variable assuming that the increased spending on poverty reduction raises output. The dashed lines indicate what these paths would be in the absence of this additional growth. So the dashed lines indicate the direct effect of debt relief with HIPC conditionality, while the solid lines indicate the combination of the direct and indirect effects. The scales for real balances, debt, aid and seignorage can all be interpreted by noting that GDP = 1. The inflation and money growth rates are expressed in decimal form.